Rational Overoptimism and Limited Liability: Online Appendix

Appendix A Motivational Evidence on Beliefs in Booms

In this section, I provide new and review existing evidence on the significance of information frictions in business cycles. First, I examine the business cycle fluctuations in forecast errors for real GDP growth by comparing the average of consensus forecast errors during booms and recessions. Forecast errors are defined as the difference between actual and the average expected GDP growth across forecasters, in the current and next three quarters, with data on forecasts taken from the Survey of Professional Forecasters. Figure A.1 shows that during booms forecast errors are positive, meaning that forecasters underestimate real output, while during recessions they overestimate it. This suggests that at the aggregate level, expectations exhibit underreaction to changes in macroeconomic quantities. Figure A.2 plots a similar measure for the annualized housing starts growth during the real estate boom-and-bust of 2006. The pattern is similar to the previous figure and suggests that forecasters underestimated housing starts growth during the boom. In my model, I show how underestimation of an increase in supply leads to an overestimation of the equilibrium market price, which may provide insight into the apparent overoptimism that fueled the housing bubble in the years preceding the crisis.

Moreover, a growing literature uses surveys of professional forecasters to measure aggregate belief stickiness, which is consistent with models of dispersed information where each agent has access to different information (Coibion and Gorodnichenko, 2012, 2015; Gemmi and Valchev, 2023).[1] Moreover, recent literature documents that firm expectations display much more disagreement than professional forecasters, about both current and future economic conditions and in dif-

[1] Bordalo et al. (2018a) provide evidence supporting behavioral overreaction in individual-level forecasts of financial and macroeconomic variables in surveys of professional forecasters. However, they still find dispersed information and belief stickiness at the consensus level. Moreover, Gemmi and Valchev (2023) presents evidence on individual survey forecasts that are inconsistent with the diagnostic expectation framework.

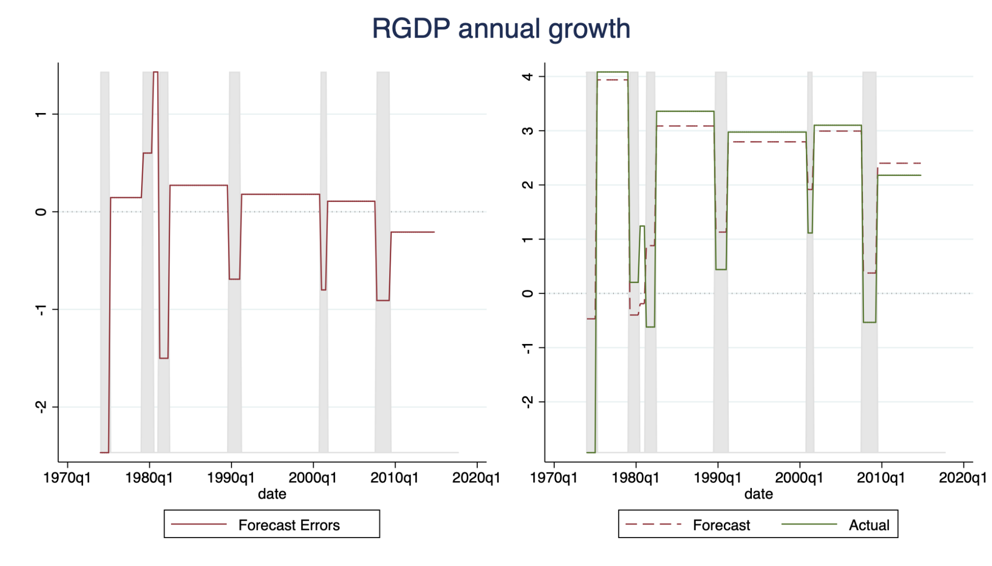

Figure A.1: Forecast errors on Real GDP growth

Notes: Left panel: the red line plots the forecast errors on annualized real GDP growth averaged in booms (non-shaded areas) and NBER recessions (shaded areas). Forecast errors are defined as $fe_t = x_t - f_t(x_t)$, where x_t is the average annualized growth of real GDP in the current and the next three quarters, and $f_t(x_t)$ the average (consensus) forecast in quarter t about annualized growth of real GDP in the current and the next three quarters. The shaded area indicates the NBER recession dates. Right panel: the dashed red line plots the average forecast on annualized real GDP growth $f_t(x_t)$, while the solid green line the actual real GDP growth x_t. All expectation data are from the Survey of Professional Forecasters, collected by the Federal Reserve's Bank of Philadelphia

ferent advanced economies (Candia et al. (2023) for a review). Coibion et al. (2018) document that managers' belief updating is consistent with the Bayesian framework, and their attention allocation to aggregates depends on incentives. Other works suggest that firms forecast about aggregates variables depends on local economic condition, consistent with models of rational inattention (Tanaka et al., 2020; Candia et al., 2021; Andrade et al., 2022; Dovern et al., 2023)

Collectively, this evidence indicates that information frictions play a crucial role in the dynamics of business cycles, which is at odds with behavioral models of overoptimism and macroeconomic models of boom-and-bust, both of which usually assume full information.[2]

[2]A prominent behavioral theory of overoptimism is belief extrapolation, and in particular diagnostic expectations, which causes agents to over-react to recent news (Gennaioli and Shleifer, 2010; Bordalo et al., 2018b, 2021).

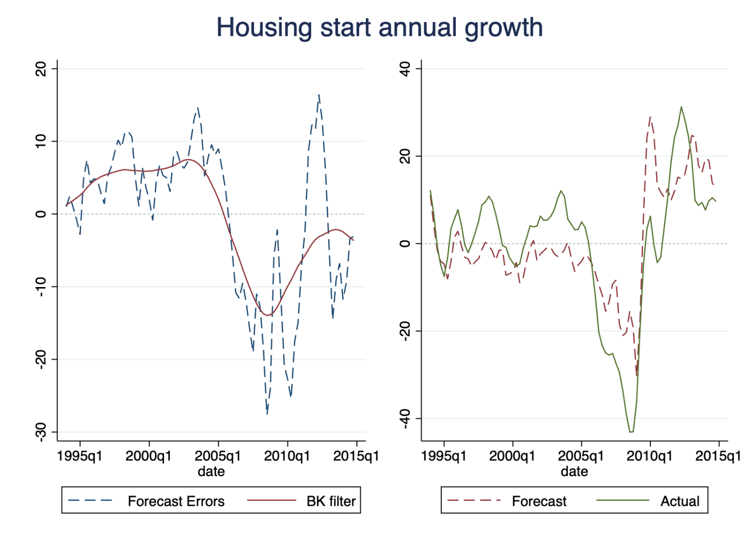

Figure A.2: Forecast errors on Housing Start

Notes: The blue line plots the forecast errors on annualized housing start growth from the Survey of Professional Forecasters, collected by the Federal Reserve's Bank of Philadelphia. Forecast errors are defined as $fe_t = x_t - f_t(x_t)$, where x_t is the average annualized growth of housing starts in the current and the next three quarters, and $f_t(x_t)$ the average (consensus) forecast in quarter t about annualized growth of housing starts in the current and the next three quarters. The red line plots the Baxter-King filtered trend, where I filtered out periods lower than 32.

Appendix B Stage-2 equilibrium

The stage-2 equilibrium can be equivalently expressed in terms of firm's issuance of bond $\tilde{b}_j$ and bond price q_j instead of loan rate r_j and loan quantity b_j, where $q_j = \frac{1}{1+r_j}$, and $\tilde{b}_j = \frac{b_j}{q_j}$.

Information. Agents observe the signal $z = \epsilon_j + \theta + \eta_j$, with $\epsilon_j \sim N(0, \sigma_\epsilon^2)$ and $\eta_j \sim N(0, \sigma_\eta^2)$, and may observe $\theta \sim N(0, \sigma_\theta^2)$. Therefore the information set of agent j is either $\Omega_j = \{z_j, \theta\}$ or $\Omega_j = \{z_j\}$ depending on whether they decide to observe aggregates in the first stage.

Define $\tilde{z} = z - \theta$. The posterior mean of the local shock is $e|\tilde{z} \sim N(E[e|\tilde{z}], Var[e|\tilde{z}])$ with $E[e|\tilde{z}] = \tilde{m}\tilde{z}$ and $\tilde{m} = \frac{\sigma_e^2}{\sigma_e^2+\sigma_\eta^2}$, while the posterior variance is $Var[e|\tilde{z}] = \frac{\sigma_e^2\sigma_\eta^2}{\sigma_e^2+\sigma_\eta^2}$. If the agent does not observe aggregates, the posterior mean of the aggregate shock is $\theta|z \sim N(E[\theta|z], Var[\theta|z])$ with $E[\theta|z] = \delta z$ and $\delta = \frac{\sigma_\theta^2}{\sigma_e^2+\sigma_\eta^2+\sigma_\theta^2}$, while the posterior variance is $Var[e|\tilde{z}] = \frac{\sigma_\theta^2(\sigma_e^2+\sigma_\eta^2)}{\sigma_e^2+\sigma_\eta^2+\sigma_\theta^2}$.

Bargaining process. Define $C(\theta) \equiv ln\left(\frac{k+\frac{1}{2}\phi k^2}{q\Lambda(M)k^\alpha}\right) - \theta$. The expected payoff of firm manager conditioning on stage-2 information set Ω_j is

$$
\begin{aligned}
E[w_{firm,j}|\Omega_j] = &- \left[1 - \int_{-\infty}^{\infty}\int_{-\infty}^{C(\theta)} \phi(\epsilon_j|\theta, z_j)d\epsilon_j\phi(\theta|\Omega_j)d\theta\right]\tilde{b}_j \\
&- \left[\int_{-\infty}^{\infty}\int_{-\infty}^{C(\theta)} \phi(\epsilon_j|\theta, z_j)d\epsilon_j\phi(\theta|\Omega_j)d\theta\right]\psi c_d k_j(q_j, \tilde{b}_j) \\
&+ k_j(q_j, \tilde{b}_j)^\alpha \int_{-\infty}^{\infty}\int_{C(\theta)}^{\infty} \Lambda(\theta)e^{\epsilon_j}\phi(\epsilon_j|\theta, z_j)d\epsilon_j e^{\theta}\phi(\theta|\Omega_j)d\theta
\end{aligned}
$$

while the expected payoff of the bank manager conditioning on stage-2 information set Ω_j is

$$
\begin{aligned}
E[w_{bank}|\Omega_j] =& b_j\left(\left[1 - \int_{-\infty}^{\infty}\int_{-\infty}^{C(\theta)} \phi(\epsilon_j|\theta, z_j)d\epsilon_j\phi(\theta|\Omega_j)d\theta\right]\right) \\
&- b_j\left(1 - \psi\left[\int_{-\infty}^{\infty}\int_{-\infty}^{C(\theta)} \phi(\epsilon_j|\theta, z_j)d\epsilon_j\phi(\theta|\Omega_j)d\theta\right]\right)\frac{q_j}{q}
\end{aligned}
\tag{B.1}
$$

where $\phi(\epsilon_j|\theta, z_j) \equiv \phi\left(\frac{C - E[\epsilon_j|\theta, z_j]}{\sqrt{Var[\epsilon_j|\theta, z_j]}}\right)$ is the posterior distribution of ϵ_j conditioning on θ and z_j, and $\phi(\theta|\Omega_j) \equiv \phi\left(\frac{\theta - E[\theta|\Omega_j]}{\sqrt{Var[\theta|\Omega_j]}}\right)$ is the posterior distribution of θ conditioning on information set Ω_j, which may or may not include θ.

Each bank and firm decide collectively the bond issuance $\tilde{b}_j$ and price q_j through Nash Bargaining

$$
\begin{aligned}
&max_{q_j,\tilde{b}_j}\ (E[w_{firm,j}|\Omega_j])^\beta(E[w_{bank,j}|\Omega_j])^{1-\beta} \\
&s.t.\ E[w_{firm}|\Omega_j] \geq 0, E[w_{bank}|\Omega_j] \geq 0
\end{aligned}
\tag{B.2}
$$

B.1 Take-it-or-leave-it offer

Assuming $\beta \to 1$, the problem becomes becomes

$$
\begin{aligned}
&max_{q_j,b_j}\ E[w_{firm,j}|\Omega_j] \\
&s.t. \quad E[w_{bank,j}|\Omega_j] \geq 0
\end{aligned}
\tag{B.3}
$$

Notice that maximizing in terms of k_j is equivalent to maximizing in terms of $\tilde{b}_j$. The resulting first-order conditions are given by

$$\begin{aligned} &E[w_{bank,j}|\Omega_j] = 0 \\ &\frac{\frac{\partial E[w_{firm,j}|\Omega_j]}{\partial \tilde{b}_j}}{\frac{\partial E[w_{bank,j}|\Omega_j]}{\partial \tilde{b}_j}} = \frac{\frac{\partial E[w_{firm,j}|\Omega_j]}{\partial q_j}}{\frac{\partial E[w_{bank,j}|\Omega_j]}{\partial q_j}} \end{aligned} \tag{B.4}$$

where each term is defined as follows. Define $pdef_j \equiv \left[\int_{-\infty}^{\infty}\int_{-\infty}^{C(\theta)} \phi(\epsilon_j|\theta, z_j)d\epsilon_j \phi(\theta|\Omega_j)d\theta\right]$. Then,

$$\begin{aligned} \frac{\partial E[w_{firm,j}|\Omega_j]}{\partial \tilde{b}_j} =& -[1 - pdef_j] - [pdef_j]\,\psi c_d \frac{\partial k_j}{\partial \tilde{b}_j} - \left[\int_{-\infty}^{\infty} \phi(C|\theta, z_j)\frac{\partial C}{\partial \tilde{b}_j} d\epsilon_j \phi(\theta|\Omega_j)d\theta\right]\psi c_d k_j \\ &+ \alpha k_j^{\alpha-1}\frac{\partial k_j}{\partial \tilde{b}_j}\int_{-\infty}^{\infty}\int_{C(\theta)}^{\infty} \Lambda(\theta)e^{\epsilon_j}\phi(\epsilon_j|\theta, z_j)d\epsilon_j e^{\theta}\phi(\theta|\Omega_j)d\theta \end{aligned} \tag{B.5}$$

where $\frac{\partial k_j}{\partial \tilde{b}_j} = \frac{q_j}{\sqrt{1+2\phi\tilde{b}_j q_j}}$, and $\frac{\partial C}{\partial \tilde{b}_j} = \frac{1}{\tilde{b}_j} - \alpha\frac{1}{k_j}\frac{\partial k_j}{\partial \tilde{b}_j}$.

$$\begin{aligned} \frac{\partial E[w_{firm,j}|\Omega_j]}{\partial \tilde{q}_j} =& -[pdef_j]\,\psi c_d \frac{\partial k_j}{\partial q_j} - \left[\int_{-\infty}^{\infty} \phi(C|\theta, z_j)\frac{\partial C}{\partial q_j} d\epsilon_j \phi(\theta|\Omega_j)d\theta\right]\psi c_d k_j \\ &+ \alpha k_j^{\alpha-1}\frac{\partial k_j}{\partial q_j}\int_{-\infty}^{\infty}\int_{C(\theta)}^{\infty} \Lambda(\theta)e^{\epsilon_j}\phi(\epsilon_j|\theta, z_j)d\epsilon_j e^{\theta}\phi(\theta|\Omega_j)d\theta \end{aligned} \tag{B.6}$$

where $\frac{\partial k_j}{\partial q_j} = \frac{\tilde{b}_j}{\sqrt{1+2\phi\tilde{b}_j q_j}}$, and $\frac{\partial C}{\partial q_j} = -\alpha\frac{1}{k_j}\frac{\partial k_j}{\partial q_j}$.

$$\frac{\partial E[w_{bank,j}|\Omega_j]}{\partial \tilde{b}_j} = \left[(1 - pdef_j) - (1 - \psi pdef_j)\frac{q_j}{q^f}\right] + \tilde{b}_j\left[-\frac{\partial pdef_j}{\partial \tilde{b}_j} + \psi\frac{q_j}{q^f}\frac{\partial pdef_j}{\partial \tilde{b}_j}\right] \tag{B.7}$$

where

$$\frac{\partial pdef_j}{\partial \tilde{b}_j} = \left[\int_{-\infty}^{\infty} \phi(C|\theta, z_j)\frac{\partial C}{\partial \tilde{b}_j} d\epsilon_j \phi(\theta|\Omega_j)d\theta\right] \tag{B.8}$$

Finally,

$$\frac{\partial E[w_{bank,j}|\Omega_j]}{\partial q_j} = +\tilde{b}_j\left[-\frac{\partial pdef_j}{\partial q_j} + \psi\frac{q_j}{q^f}\frac{\partial pdef_j}{\partial q_j} - (1-\psi pdef_j)\frac{1}{q^f}\right] \tag{B.9}$$

where

$$\frac{\partial pdef_j}{\partial q_j} = \left[\int_{-\infty}^{\infty}\phi(C|\theta, z_j)\frac{\partial C}{\partial q_j}d\epsilon_j\phi(\theta|\Omega_j)d\theta\right] \tag{B.10}$$

B.2 General problem

Allowing for bargaining power $0 < \beta < 1$, the FOCs are (assuming the non-negative expected profit constraints do not bind)

$$\begin{aligned}\beta(E[w_{firm,j}|\Omega_j])^{-1}\frac{\partial E[w_{firm,j}|\Omega_j]}{\partial\tilde{b}_j} + (1-\beta)(E[w_{bank,j}|\Omega_j])^{-1}\frac{\partial E[w_{bank,j}|\Omega_j]}{\partial\tilde{b}_j} = 0\\ \beta(E[w_{firm,j}|\Omega_j])^{-1}\frac{\partial E[w_{firm,j}|\Omega_j]}{\partial q_j} + (1-\beta)(E[w_{bank,j}|\Omega_j])^{-1}\frac{\partial E[w_{bank,j}|\Omega_j]}{\partial q_j} = 0\end{aligned} \tag{B.11}$$

Appendix C Proofs

Proposition 1. Assume no limited liability, no default cost, and no investment adjustment cost $c_d = \psi = \phi = 0$. To simplify the exposition, I drop the subscript j. Use the definition of $q = \frac{1}{1+r}$ and $q\tilde{b} = k$. As a result, $C = \left(\frac{k^{1-\alpha}}{q\Lambda(\theta)}\right) - \theta$.

Foc 1. Consider the first first-order condition in (B.4).

$$q = q^f\left[1 - \int_{-\infty}^{\infty}\Phi_e(C(\theta)|z,\theta)\phi_\theta(\theta|\Omega)d\theta\right] \tag{C.1}$$

In steady state

$$q^* = q^f\left[1 - \Phi\left(\frac{C^*}{\sqrt{Var[\epsilon|\theta, z]}}\right)\right] \tag{C.2}$$

where x^* is the steady state value of variable x. Differentiating

$$dq = -\, q^f \Phi\left(\frac{C^*}{\sqrt{Var[\epsilon|\theta,z]}}\right) \int_{-\infty}^{\infty} \left[dC - dE[e|z,\theta]\right] \phi_\theta\left(\theta|z\right) d\theta \tag{C.3}$$

where $dC = (1-\alpha)]\hat{k} - \hat{q} - (\eta_{\Lambda(M),\theta} - 1)d\theta$, where $\eta_{\Lambda(M),\theta} \equiv -\frac{1}{\Lambda(M)}\Lambda'(M)M'(\theta)$, and $dE[\epsilon|z,\theta] = \frac{\partial E[\epsilon|\tilde{z}]}{\partial\theta}d\theta + \frac{\partial E[\epsilon|\tilde{z}]}{\partial z}dz$. Therefore

$$dq = -q^f \Phi\left(\frac{C^*}{\sqrt{Var[\epsilon|\theta,z]}}\right) \int_{-\infty}^{\infty} \left[(1-\alpha)]\hat{k} - \hat{q} - \eta_{\Lambda(M),\theta}d\theta - \frac{\partial E[\epsilon|\tilde{z}]}{\partial\theta}d\theta - \frac{\partial E[\epsilon|\tilde{z}]}{\partial z}dz\theta\right] \phi_\theta\left(\theta|z\right) d\theta \tag{C.4}$$

Denote $a \equiv ln(A)$ and notice that

$$\begin{aligned} E[a|z,\theta] &= \tilde{m}(z-\theta) + \theta \\ &= \frac{\partial E[\epsilon|\tilde{z}]}{\partial z}z + \frac{\partial E[\epsilon|\tilde{z}]}{\partial\theta}\theta + \theta \end{aligned}$$

Moreover, $\hat{M} \equiv \frac{dM}{M} = \frac{M'(\theta)d\theta}{M}$ and therefore

$$\begin{aligned} \eta_{\Lambda(M),\theta}d\theta &= -\frac{M}{\Lambda(M)}\Lambda'(M)\frac{M'(\theta)d\theta}{M} \\ &= \eta_{\Lambda,M}\hat{M} \end{aligned} \tag{C.5}$$

where $\eta_{\Lambda,M} \equiv \frac{\nu-\xi}{1-(1-\alpha)\xi}$. Substitute back and divide by steady-state value

$$\hat{q} = \tilde{L}_1\left\{-\left(1-\alpha\right)\hat{k} + E[a|z] + \eta_{\Lambda,M}E[\hat{M}|z]\right\} \tag{C.6}$$

where $\tilde{L}_1 = \frac{\phi\left(\frac{C^*}{\sqrt{Var[\epsilon|\theta,z]}}\right)}{\left[1-\Phi\left(\frac{C^*}{\sqrt{Var[\epsilon|\theta,z]}}\right)-\phi\left(\frac{C^*}{\sqrt{Var[\epsilon|\theta,z]}}\right)\right]}$.

Foc 2. Differentiate the second first-order condition in (B.4)

$$\frac{d\frac{\partial E[w_{firm,j}|\Omega_j]}{\partial \tilde{b}_j}}{\frac{\partial E[w_{firm,j}|\Omega_j]}{\partial \tilde{b}_j}} - \frac{d\frac{\partial E[w_{bank,j}|\Omega_j]}{\partial \tilde{b}_j}}{\frac{\partial E[w_{bank,j}|\Omega_j]}{\partial \tilde{b}_j}} = \frac{d\frac{\partial E[w_{firm,j}|\Omega_j]}{\partial q_j}}{\frac{\partial E[w_{firm,j}|\Omega_j]}{\partial q_j}} - \frac{d\frac{\partial E[w_{bank,j}|\Omega_j]}{\partial q_j}}{\frac{\partial E[w_{bank,j}|\Omega_j]}{\partial q_j}} \tag{C.7}$$

and consider each term individually.

- From equation (B.5), the derivative of the expected firm's payoff with respect to bond $\tilde{b}$ is given by

$$\begin{aligned}\frac{\partial E[w_{firm}|\Omega]}{\partial \tilde{b}} = &-\left[1 - \int_{-\infty}^{\infty} \Phi_e(C(\theta)|z,\theta)\phi_\theta(\theta|\Omega)d\theta\right] \\ &+ \alpha k_j^{\alpha-1} q \int_{-\infty}^{\infty} \Lambda(\theta) e^{\frac{Var[\epsilon|\theta,z]}{2}+E[\epsilon|\theta,z]} \Phi_\epsilon\left(\frac{Var[\epsilon|\theta,z]+E[\epsilon|\theta,z]-C(\theta)}{\sqrt{Var[\epsilon|\theta,z]}}\right) e^\theta \phi(\theta|\Omega_j)d\theta\end{aligned}$$

Differentiating,

$$\begin{aligned}d\frac{\partial E[w_{firm}|\Omega]}{\partial \tilde{b}} =&\phi_e\left(\frac{C^*}{\sqrt{Var[\epsilon|\theta,z]}}\right)\left\{\hat{q} - (1-\alpha)\,\hat{k} + E[a|z] + \eta_{\Lambda,M}E[\hat{M}|z]\right\} \\ &+ \alpha k_j^{\alpha-1} q \Lambda e^{\frac{Var[\epsilon|\theta,z]}{2}} \Phi_\epsilon\left(\cdot\right)\left\{\hat{q} - (1-\alpha)\,\hat{k} + E[a|z] + \eta_{\Lambda,M}E[\hat{M}|z]\right\} \\ &- \alpha k_j^{\alpha-1} q \Lambda e^{\frac{Var[\epsilon|\theta,z]}{2}} \phi_\epsilon\left(\cdot\right)\left\{-\hat{q} + (1-\alpha)\,\hat{k} - E[a|z] - \eta_{\Lambda,M}E[\hat{M}|z]\right\} \\ =&\left\{\alpha k_j^{\alpha-1} q \Lambda e^{\frac{Var[\epsilon|\theta,z]}{2}}\left[\Phi_\epsilon\left(\cdot\right)+\phi_\epsilon\left(\cdot\right)\right] + \phi_e\left(\frac{C^*}{\sqrt{Var[\epsilon|\theta,z]}}\right)\right\}\times \\ &\times\left\{\hat{q} - (1-\alpha)\,\hat{k} + E[a|z] + \eta_{\Lambda,M}E[\hat{M}|z]\right\}\end{aligned} \tag{C.8}$$

As a result,

$$\begin{aligned}\frac{d\frac{\partial E[w_{firm,j}|\Omega_j]}{\partial \tilde{b}_j}}{\frac{\partial E[w_{firm,j}|\Omega_j]}{\partial \tilde{b}_j}} =& \frac{\left\{\alpha k_j^{\alpha-1} q \Lambda e^{\frac{Var[\epsilon|\theta,z]}{2}}\left[\Phi_\epsilon\left(\cdot\right)+\phi_\epsilon\left(\cdot\right)\right] + \phi_e\left(\frac{C^*}{\sqrt{Var[\epsilon|\theta,z]}}\right)\right\}}{\frac{\partial E[w_{firm,j}|\Omega_j]}{\partial \tilde{b}_j}}\times \\ &\times\left\{\hat{q} - (1-\alpha)\,\hat{k} + E[a|z] + \eta_{\Lambda,M}E[\hat{M}|z]\right\} \\ =& L_1\left\{\hat{q} - (1-\alpha)\,\hat{k} + E[a|z] + \eta_{\Lambda,M}E[\hat{M}|z]\right\}\end{aligned} \tag{C.9}$$

where $L_1 \equiv \frac{\left\{\alpha k_j^{\alpha-1} q\Lambda e^{\frac{Var[\epsilon|\theta,z]}{2}}[\Phi_\epsilon(\cdot)+\phi_\epsilon(\cdot)]+\phi_e\left(\frac{C^*}{\sqrt{Var[\epsilon|\theta,z]}}\right)\right\}}{\frac{\partial E[w_{firm,j}|\Omega_j]}{\partial \tilde{b}_j}}$.

- From equation (B.6), the derivative of the expected firm's payoff with respect to bond price q is given by

$$\frac{\partial E[d_{firm}|\Omega]}{\partial q} = \alpha k_j^{\alpha-1}\frac{k}{q}\int_{-\infty}^{\infty}\Lambda(\theta)e^{\frac{Var[\epsilon|\theta,z]}{2}+E[\epsilon|\theta,z]}\Phi_\epsilon\left(\frac{Var[\epsilon|\theta,z]+E[\epsilon|\theta,z]-C(\theta)}{\sqrt{Var[\epsilon|\theta,z]}}\right)e^\theta\phi(\theta|\Omega_j)d\theta \tag{C.10}$$

Differentiating,

$$\begin{aligned} d\frac{\partial E[d_{firm}|\Omega]}{\partial q} =& \alpha k_j^{\alpha-1}\frac{k}{q}\Lambda e^{\frac{Var[\epsilon|\theta,z]}{2}}\left[\Phi_\epsilon\left(\cdot\right)+\phi_\epsilon\left(\cdot\right)\right]\left\{\hat{q}-\left(1-\alpha\right)\hat{k}+E[a|z]+\eta_{\Lambda,M}E[\hat{M}|z]\right\} \\ & + \alpha k_j^{\alpha-1}\frac{k}{q}\Lambda e^{\frac{Var[\epsilon|\theta,z]}{2}}\Phi_\epsilon\left(\cdot\right)(\hat{k}-2\hat{q}) \end{aligned} \tag{C.11}$$

therefore

$$\begin{aligned} \frac{d\frac{\partial E[w_{firm,j}|\Omega]}{\partial q}}{\frac{\partial E[w_{firm,j}|\Omega]}{\partial q}} &= \frac{\alpha k_j^{\alpha-1}\frac{k}{q}\Lambda e^{\frac{Var[\epsilon|\theta,z]}{2}}\left[\Phi_\epsilon\left(\cdot\right)+\phi_\epsilon\left(\cdot\right)\right]}{\frac{\partial E[w_{firm,j}|\Omega]}{\partial q}}\times \\ &\times\left\{\hat{q}-\left(1-\alpha\right)\hat{k}+E[a|z]+\eta_{\Lambda,M}E[\hat{M}|z]\right\}+(\hat{k}-2\hat{q}) \\ &= L_2\left\{\hat{q}-\left(1-\alpha\right)\hat{k}+E[a|z]+\eta_{\Lambda,M}E[\hat{M}|z]\right\}+(\hat{k}-2\hat{q}) \end{aligned} \tag{C.12}$$

where $L_2 \equiv \frac{\alpha k_j^{\alpha-1}\frac{k}{q}\Lambda e^{\frac{Var[\epsilon|\theta,z]}{2}}[\Phi_\epsilon(\cdot)+\phi_\epsilon(\cdot)]}{\frac{\partial E[w_{firm,j}|\Omega]}{\partial q}}$.

- From equation (B.7), the derivative of the expected bank's payoff with respect to bond $\tilde{b}_j$ is given by

$$\begin{aligned} \frac{\partial E[d_{bank}|\Omega^i]}{\partial \tilde{b}} = &\left[\left(1-\int_{-\infty}^{\infty}\Phi_e(C(\theta)|z,\theta)\phi_\theta(\theta|\Omega)d\theta\right)-\frac{q}{q^f}\right] \\ &-(1-\alpha)\int_{-\infty}^{\infty}\phi_e(C(\theta)|z,\theta)\phi_\theta(\theta|\Omega)d\theta \end{aligned}$$

Differentiating,

$$d\frac{\partial E[d_{bank}|\Omega]}{\partial \tilde{b}} = -\phi_e\left(\frac{C^*}{\sqrt{Var[\epsilon|\theta,z]}}\right)\left\{\hat{q}-(1-\alpha)\,\hat{k}+E[a|z]+\eta_{\Lambda,M}E[\hat{M}|z]\right\}$$
$$-\frac{q}{q^f}\hat{q}-(1-\alpha)\phi_e\left(\frac{C^*}{\sqrt{Var[\epsilon|\theta,z]}}\right)\frac{C^*}{Var[\epsilon|\theta,z]}\left\{\hat{q}-(1-\alpha)\,\hat{k}+E[a|z]+\eta_{\Lambda,M}E[\hat{M}|\right.$$
(C.13)

therefore

$$\frac{d\frac{\partial E[d_{bank}|\Omega]}{\partial \tilde{b}}}{\frac{\partial E[d_{bank}|\Omega]}{\partial \tilde{b}}} = \frac{\phi_e\left(\frac{C^*}{\sqrt{Var[\epsilon|\theta,z]}}\right)(1+(1-\alpha)\frac{C}{Var[\epsilon|\theta,z]})}{\frac{\partial E[d_{bank}|\Omega]}{\partial \tilde{b}}}\left\{\hat{q}-(1-\alpha)\,\hat{k}+E[a|z]+\eta_{\Lambda,M}E[\hat{M}|z]\right\}$$
$$-\frac{\frac{q}{q^f}}{\frac{\partial E[d_{bank}|\Omega]}{\partial \tilde{b}}}\hat{q}$$
$$= L_3\left\{\hat{q}-(1-\alpha)\,\hat{k}+E[a|z]+\eta_{\Lambda,M}E[\hat{M}|z]\right\}-L_4\hat{q}$$
(C.14)

where $L_3 \equiv \frac{\phi_e\left(\frac{C^*}{\sqrt{Var[\epsilon|\theta,z]}}\right)(1+(1-\alpha)\frac{C}{Var[\epsilon|\theta,z]})}{\frac{\partial E[d_{bank}|\Omega]}{\partial \tilde{b}}}\left\{\hat{q}-(1-\alpha)\,\hat{k}+E[a|z]+\eta_{\Lambda,M}E[\hat{M}|z]\right\}$ and $L_4 \equiv \frac{\frac{q}{q^f}}{\frac{\partial E[d_{bank}|\Omega]}{\partial \tilde{b}}}$.

- From equation (B.9), the derivative of the expected bank's payoff with respect to bond price q_j is given by

$$\frac{\partial E[d_{bank}|\Omega]}{\partial q} = \frac{k}{q}\left[\alpha\int_{-\infty}^{\infty}\phi_e(C(\theta)|z,\theta)\phi_\theta(\theta|\Omega)d\theta\frac{1}{q}-\frac{1}{q^f}\right]$$

differentiating,

$$
\begin{aligned}
d\frac{\partial E[d_{bank}|\Omega]}{\partial q} &= \frac{k}{q}(\hat{k}-\hat{q})\left[\alpha\phi_e\left(\frac{C^*}{\sqrt{Var[\epsilon|\theta,z]}}\right)\frac{1}{q}-\frac{1}{q^f}\right]+ \\
&+\frac{k}{q}\alpha\phi_e\left(\frac{C^*}{\sqrt{Var[\epsilon|\theta,z]}}\right)\frac{C^*}{Var[\epsilon|\theta,z]}\frac{1}{q}\left\{\hat{q}-(1-\alpha)\,\hat{k}+E[a|z]+\eta_{\Lambda,M}E[\hat{M}|z]\right\} \\
&-\frac{k}{q}\alpha\phi_e\left(\frac{C^*}{\sqrt{Var[\epsilon|\theta,z]}}\right)\frac{1}{q}\hat{q}
\end{aligned}
$$

Therefore

$$
\begin{aligned}
\frac{d\frac{\partial E[d_{bank}|\Omega]}{\partial q}}{\frac{\partial E[d_{bank}|\Omega]}{\partial q}} &= (\hat{k}-\hat{q})-\frac{\frac{k}{q}\alpha\phi_e\left(\frac{C^*}{\sqrt{Var[\epsilon|\theta,z]}}\right)}{\frac{\partial E[d_{bank}|\Omega]}{\partial q}}\hat{q} \\
&-\frac{\frac{k}{q}\alpha\phi_e\left(\frac{C^*}{\sqrt{Var[\epsilon|\theta,z]}}\right)\frac{C^*}{Var[\epsilon|\theta,z]}}{\frac{\partial E[d_{bank}|\Omega]}{\partial q}}\left\{\hat{q}-(1-\alpha)\,\hat{k}+E[a|z]+\eta_{\Lambda,M}E[\hat{M}|z]\right\} \\
&= (\hat{k}-\hat{q})-L_5\left\{\hat{q}-(1-\alpha)\,\hat{k}+E[a|z]+\eta_{\Lambda,M}E[\hat{M}|z]\right\}-L_6\hat{q}
\end{aligned}
\tag{C.15}
$$

where $L_5 = \frac{\frac{k}{q}\alpha\phi_e\left(\frac{C^*}{\sqrt{Var[\epsilon|\theta,z]}}\right)\frac{C^*}{Var[\epsilon|\theta,z]}}{\frac{\partial E[d_{bank}|\Omega]}{\partial q}}$ and $L_6 = \frac{\frac{k}{q}\alpha\phi_e\left(\frac{C^*}{\sqrt{Var[\epsilon|\theta,z]}}\right)}{\frac{\partial E[d_{bank}|\Omega]}{\partial q}}$.

Finally, substitute equations (C.9), (C.12), (C.14), and (C.15) in equation (C.7) and get

$$
\begin{aligned}
(L_1-L_2-L_3-L_5+L_4+1-L_6)\hat{q} &= -(L_1-L_2-L_3-L_5)\left\{-(1-\alpha)\,\hat{k}+E[a|z]+\eta_{\Lambda,M}E[\hat{M}|z]\right\} \\
\hat{q} &= \tilde{L}_2\left\{-(1-\alpha)\,\hat{k}+E[a|z]+\eta_{\Lambda,M}E[\hat{M}|z]\right\}
\end{aligned}
\tag{C.16}
$$

where $\tilde{L}_2 \equiv \frac{-(L_1-L_2-L_3-L_5)}{(L_1-L_2-L_3-L_5+L_4+1-L_6)}$.

Equilibrium. Substitute equation (C.6) in (C.16)

$$\tilde{L}_1 \left\{ -\left(1-\alpha\right)\hat{k} + E[a|z] + \eta_{\Lambda,M} E[\hat{M}|z] \right\} = \tilde{L}_2 \left\{ -\left(1-\alpha\right)\hat{k} + E[a|z] + \eta_{\Lambda,M} E[\hat{M}|z] \right\}$$
$$(\tilde{L}_1 - \tilde{L}_2) \left\{ -\left(1-\alpha\right)\hat{k} + E[a|z] + \eta_{\Lambda,M} E[\hat{M}|z] \right\} = 0 \tag{C.17}$$

therefore, the stage-2 equilibrium k and q are given by

$$\begin{aligned} \hat{k} &= \frac{1}{1-\alpha}(E[a|z] - \gamma E[\hat{M}|z]) \\ \hat{q} &= 0 \end{aligned} \tag{C.18}$$

Where $\gamma \equiv -\eta_{\Lambda(M),M} = -\frac{\nu-\xi}{1-(1-\alpha)\xi}$. If $\nu < \xi$, then $\gamma > 0$. Therefore $\hat{r}_j \propto \hat{q} = 0$.

Since $M = \left\{ \left[\frac{w}{(1-\alpha)\xi\nu}\right]^{\frac{(1-\alpha)}{(1-\alpha)\xi-1}} \left[\int^N A_j k_j^\alpha dj\right]^{\frac{1}{\xi}} \right\}^{\frac{1-(1-\alpha)\xi}{1-(1-\alpha)\nu}}$, log deviation of M around the stochastic steady state equals

$$\hat{M} = \mu(\alpha \hat{K} + \theta)$$

where $\mu \equiv \frac{1}{\xi}\frac{1-(1-\alpha)\xi}{1-(1-\alpha)\nu} > 0$ and $\hat{K} = \int^j k_j dj$. One can write

$$\begin{aligned} \hat{k} &= \frac{1}{1-\alpha}(E[a|z] - \gamma\mu E[\theta + \alpha\hat{K}|z]) \\ \hat{q} &= 0 \end{aligned}$$

Moreover, from (C.1)

$$\hat{q}_j = -\zeta \hat{p}(def_j|\Omega_j) = 0 \tag{C.19}$$

where $\zeta \equiv \frac{p^*(def|0)}{1-p^*(def|0)}$.

The expected level deviation of bank j's profit from steady state conditioning on state θ equals

$$\begin{aligned} E[w_{bank,j}|z_j,\theta] &= -p^*(def|0)\hat{p}(def_j|z_j,\theta) - \frac{q^*}{q_j}\hat{q}_j \\ &= -p^*(def|0)[\hat{p}(def_j|z_j,\theta) - E[\hat{p}(def_j|\Omega_j)|\theta]] \end{aligned} \tag{C.20}$$

which is zero for each θ if $\theta \in \Omega_j$. ■

Proposition 2. Consider the global game when θ is observed

$$\hat{k} = \frac{1}{1-\alpha} E[a_j|z] - \frac{1}{1-\alpha}\gamma\mu\left(\theta + \alpha\hat{K}\right) \tag{C.21}$$

where $E[a_j|z_j, \theta] = \tilde{m}(z_j - \theta) + \theta$, where $z_j = a_j + \eta_j$ and $\tilde{m} = \frac{\sigma_e^2}{\sigma_e^2+\sigma_\eta^2}$. Aggregating across islands

$$\begin{aligned} K &= \frac{1}{1-\alpha}(1-\gamma\mu)\theta - \frac{\alpha}{1-\alpha}\gamma\mu K \\ K &= \frac{(1-\gamma\mu)}{1-\alpha+\alpha\gamma\mu}\theta \end{aligned} \tag{C.22}$$

■

Proposition 3. Consider the global game when θ is not observed

$$\hat{k} = \frac{1}{1-\alpha} E[a_j|z_j] - \frac{1}{1-\alpha}\gamma\mu\left(E[\hat{\theta}|z_j] + \alpha E[\hat{K}|z_j]\right) \tag{C.23}$$

where $E[a_j|z_j] = mz_j$, with $m = \frac{\sigma_e^2+\sigma_\theta^2}{\sigma_e^2+\sigma_\theta^2+\sigma_\eta^2}$, and $E[\theta|z_j] = \delta z_j$ where $\delta = \frac{\sigma_e^2}{\sigma_e^2+\sigma_\theta^2+\sigma_\eta^2}$. Following Morris and Shin (2002), I guess the linear solution $k_j = \chi z_j$

$$\begin{aligned} k_j &= \frac{1}{1-\alpha}(m - \gamma\mu[1+\alpha\chi]\delta)z_j \\ \chi &= \frac{1}{1-\alpha}(m - \gamma\mu[1+\alpha\chi]\delta) \\ \chi &= \frac{(m-\gamma\mu\delta)}{1-\alpha+\gamma\mu\alpha\delta} \\ K &= \frac{(m-\gamma\mu\delta)}{1-\alpha+\gamma\mu\alpha\delta}\theta \end{aligned} \tag{C.24}$$

■

Corollary 4. The loglinearized individual revenues $\hat{\pi}_j$ if $\theta \notin \Omega_j$ equals

$$\begin{aligned} \widehat{\pi}_j &= -\gamma\hat{M} + a_j + \alpha k_j \\ &= -\gamma\mu\left(\theta + \alpha\frac{(m-\gamma\mu\delta)}{1-\alpha+\gamma\mu\alpha\delta}\theta\right) + a_j + \alpha k_j \end{aligned} \tag{C.25}$$

Since $E[a_j|z_j] = mz_j$ and $E[\theta|z_j] = \delta z_j$,

$$
\begin{aligned}
E[\widehat{\pi}_j|z_j,\theta] - E[E[\widehat{\pi}_j|z_j]|\theta] =& E[a_j|z_j,\theta] - E[E[a_j|z_j]|\theta] - \gamma(\hat{M} - E[\hat{M}|z_j]) \\
=& \left[(1-m) - \gamma\mu(1-\delta)\left(1+\alpha\frac{(m-\gamma\mu\delta)}{1-\alpha+\gamma\mu\alpha\delta}\right)\right]\theta
\end{aligned}
\tag{C.26}
$$

It implies that average forecast errors are a positive function of θ if

$$
\begin{aligned}
(1-m) - \gamma\mu(1-\delta)\left(1+\alpha\frac{(m-\gamma\mu\delta)}{1-\alpha+\gamma\mu\alpha\delta}\right) > 0 \\
(m-\gamma\mu\delta)\,(1-\alpha+\alpha\gamma\mu) > (1-\gamma\mu)(1-\alpha+\gamma\mu\alpha\delta)
\end{aligned}
\tag{C.27}
$$

■

Corollary 5. Consider the actual probability of default of firm j in dispersed information conditioning on aggregate shock θ: $p(def_j|z_j,\theta) \equiv \Phi_{e|\tilde{z}}(C(\theta))$. The first-order approximation around the risky steady state is

$$
\hat{p}(def_j|z_j,\theta) = \frac{\phi_{e|0}(C^*)}{\Phi_{e_0}(C^*)}\left[(1-\alpha)\,\hat{k}_j - \hat{q}_j + \gamma\hat{M} - E[a_j|z_j,\theta]\right]
\tag{C.28}
$$

Aggregating across islands

$$
\begin{aligned}
\hat{p}(def|z_j,\theta) &= \xi\left[(1-\alpha)\,\hat{K} - \hat{Q} + \gamma\hat{M} - \theta\right] \\
\hat{p}(def|z_j,\theta) &= \xi\left[(1-\alpha+\alpha\gamma\mu)\,\hat{K} - (1-\gamma\mu)\theta\right] \\
\hat{p}(def|z_j,\theta) &= \xi\left[(1-\alpha+\alpha\gamma\mu)\,\frac{(m-\gamma\mu\delta)}{1-\alpha+\gamma\mu\alpha\delta} - (1-\gamma\mu)\right]\theta
\end{aligned}
\tag{C.29}
$$

Then it implies that $\frac{\partial\hat{p}(def|\theta)}{\partial\theta} > 0$ if

$$
(m-\gamma\mu\delta)\,(1-\alpha+\alpha\gamma\mu) > (1-\gamma\mu)(1-\alpha+\gamma\mu\alpha\delta)
\tag{C.30}
$$

■

Corollary 6. Consider the log deviation of the perceived probability of default from the steady state, that is conditioning on info set $\Omega_j = \{z_j\}$.

$$\hat{p}(def_j|z_j) = \frac{\phi_{e|0}(C^*)}{\Phi_{e_0}(C^*)}\left[(1-\alpha)\,\hat{k}_j - \hat{q}_j + \gamma E[\hat{M}|z_j] - E[a_j|z_j]\right] \tag{C.31}$$

Consider the log deviation of the actual probability of default from steady state, meaning conditioning on info set $\Omega_j = \{z_j, \theta\}$.

$$\hat{p}(def_j|z_j,\theta) = \frac{\phi_{e|0}(C^*)}{\Phi_{e_0}(C^*)}\left[(1-\alpha)\,\hat{k}_j - \hat{q}_j + \gamma\hat{M} - E[a_j|z_j,\theta]\right] \tag{C.32}$$

The average bank profits equal the difference between the two

$$\begin{aligned} E[\tilde{\pi}_{bank,j}|z_j,\theta] &\propto -[\hat{p}(def_j|z_j,\theta) - E[\hat{p}(def_j|z_j)|\theta]] \\ &\propto -[E[a_j|z_j,\theta] - E[E[a_j|z_j]|\theta] - \gamma(M - E[M|z_j])] \end{aligned} \tag{C.33}$$

from the proof of corollary 4, it follows that average bank profits are a negative function of θ if

$$(m - \gamma\mu\delta)\,(1-\alpha+\alpha\gamma\mu) > (1-\gamma\mu)(1-\alpha+\gamma\mu\alpha\delta) \tag{C.34}$$

■

Appendix D Discussion of Corollary 3

As stated in Corollary 3, the difference in aggregate investment in dispersed information and full information depends positively on θ, and therefore the information friction leads to an amplification of credit booms if

$$(m - \gamma\mu\delta)(1-\alpha+\gamma\mu\alpha) > (1-\gamma\mu)(1-\alpha+\gamma\mu\alpha\delta) \tag{D.1}$$

Otherwise, information frictions lead to a dampening of credit booms.

Intuitively, not observing θ leads to underestimating both the PE effect and GE effect, with opposite effects on optimal investment. Whether investment is higher with dispersed information than with full information depends on how much observing aggregates increases (i) posterior beliefs about local productivity (PE) and (ii) posterior beliefs about aggregate intermediate output (GE). First, suppose the signal z_j is infinitely noisy, $\sigma_\eta \to \infty$, then $m = \delta = 0$ and condition (D.1) is not satisfied. The intuition is as follows. Without signals on local productivity, the aggregate shock is the only source of information. If agents do not observe this shock either, investment in all states is equal to the steady state level. If agents are instead able to observe it, a higher aggregate shock θ increases their posterior beliefs on both local technology (PE) and aggregate investment (GE), but the only existing equilibrium is one in which the first outweighs the second and optimal local investment increases.[3] Second, suppose the signal z_j is noiseless, $\sigma_\eta \to 0$, then $m = 1, \delta < 1$ and condition (D.1) is satisfied. In this case, agents observe local productivity perfectly, independent of their information about the aggregate shock. However, observing aggregates provides information about the investment decisions of the other firms, and hence about the negative endogenous GE effect. In the dispersed information setting, agents underestimate the increase in competition after an aggregate shock and over-invest relative to the economy with informed agents.

Appendix E Manager compensation

Suppose we interpret the limited liability constraint as resulting from a manager's convex compensation structure. The structure is as follows:

$$w_j = \begin{cases} (1-\psi)d_j + \psi(d_j - \tilde{P}) & \text{if } d_j \geq \tilde{P} \\ (1-\psi)d_j & \text{if } d_j < \tilde{P} \end{cases} \tag{E.1}$$

[3] To see this, suppose that the negative GE effect from higher aggregate investment was stronger than the positive PE effect from higher local technology, and optimal local investment fell in θ. Aggregate investment would then be inversely proportional to θ, and the GE force would be positive, not negative, for the island, leading to a contradiction.

where d_j is the company's payoff, bank or firm, and $\tilde{P}$ is the profit level corresponding to the exercise price of the manager's options. The larger the amount of options in the manager's compensation scheme ψ, the lower his exposure to the company's losses and therefore higher his insurance against the company's losses. Assume for simplicity that $\tilde{P} = 0$, meaning that the manager's options are in the money when the profits of the firm are positive, i.e. in the non-default state. Therefore the payoff structure is equivalent to in Section 2.1.

A more general compensation structure would consist of β_m shares of company's equity, of which ψ are options.

$$w_j = \begin{cases} \beta_m(1-\psi)d_j + \beta_m\psi(d_j - \tilde{P}) & \text{if } d_j \geq \tilde{P} \\ \beta_m(1-\psi)d_j & \text{if } d_j < \tilde{P} \end{cases} \tag{E.2}$$

The net profits for the shareholder are $(1-\beta_m)d_j$ if the profit is positive and $\beta_m\psi d_j$ otherwise. In particular, $\beta_m < 1$ ensures a positive expected leftover profit for the shareholders. However, setting $\beta_m = 1$ does not affect qualitatively the results. Moreover, an additional fixed compensation $\bar{w}$ would not affect the manager's incentives and therefore his decisions.

Appendix F Calibration

First, I set $\xi = 0.833$ to match a markup of 20%, which is inside the set of values estimated in the macro literature (for a review, see Basu (2019)). Together with a capital share $\tilde{\alpha} = 0.33$, it implies $\alpha = \frac{\tilde{\alpha}\xi}{1-(1-\tilde{\alpha})\xi} = 0.624$. The return to scale of final good producer ν can be expressed similarly as a function of the final good sector markup and the intermediate good share in production. Assuming the latter equals 0.5 (approximately the average value for the US economy over a long period) and a markup of 50% gives $\nu = 0.5$. The larger markup in the retail and wholesale sectors with respect to other sectors is in line with the evidence in De Loecker et al. (2020). However, the condition $\nu < \xi$ would be satisfied by any final good sector markup larger than 13%.[4]

[4] Assume the final good sector faces a demand given by $P = Y^{\tilde{\xi}-1}$ and have a production function $Y = M^{\tilde{\nu}}X^{1-\tilde{\nu}}$, where X is some other variable input. After maximizing X out, the profit function would be proportional to $\pi \propto$

Since TFP in my model is i.i.d., I set the aggregate volatility equal to the unconditional volatility implied by a standard autoregressive process with quarterly shock volatility 0.02 and autoregressive coefficient 0.995, which gives $\sigma_\theta = 0.2$. I set the idiosyncratic TFP volatility $\sigma_e = 3\sigma_\theta$, where the ratio 3 is somewhere between the macro structural estimates (e.g. ≈ 15, Maćkowiak and Wiederholt (2015)) and the micro empirical estimates (e.g. ≈ 1.1, Castro et al. (2015)). Moreover, I set the private noise $\sigma_\eta = \sigma_a$, where σ_a is the total volatility of TFP. Because the model aims to capture low-frequency credit boom&busts as in the macro-finance empirical literature, I set the risk-free rate to the 5-year implied return from a one-year T-bill of 2%, which gives $r^f = 0.1$. The corporate tax rate is set to 20% (CBO, 2017).

While In section 3.1, I abstract from limited liability and set $\psi = 0$, in section 4, I perform comparative statics on this parameter to study how risk-taking incentives affect lending and information choice. Finally, I calibrate the cost of information c such that with no limited liability, it is optimal for all islands to collect information ($\lambda = 1$), a cost that corresponds to around 3% of the firm's dividends in the full information economy.

Appendix G Infinite-period extension

I extend the model to an infinite-period setting to compare its predictions to the existing evidence on credit cycles. First, I review the existing evidence on the paths of spreads and credit before financial crises, and then I compare the performance of my model to the data. While a full quantitative estimation of the model is beyond the scope of this paper, I demonstrate that the model with a standard calibration can generate realistic boom-and-bust dynamics.

I focus on financial crises, defined by the literature "as events during which a country's banking sector experiences bank runs, sharp increases in default rates accompanied by large losses of capital that result in public intervention, bankruptcy, or forced merger of financial institutions" (Jordà et al., 2013). I compare my model to two sets of evidence from Krishnamurthy and Li (2021):

$M^{\frac{\tilde{\nu}\tilde{\xi}}{1-(1-\tilde{\nu})\tilde{\xi}}} \equiv M^{\nu}$. Given an intermediate share of $\tilde{\nu} = 0.5$ and $\xi = 0.833$, the condition $\nu < \xi$ implies a final good sector markup $\frac{1}{\xi} > 1.13$.

first, the pre-crisis path of spreads and credit; second, the predictive power of spreads and credit growth in forecasting financial crises.

Pre-crisis period. Conditioning on a crisis at time t, consider the path of spreads and credit in the 5 years preceding the crisis. First, credit spreads are 0.34σs below their country mean, where the mean is defined to exclude the crisis and the 5 years after the crisis. Second, credit/GDP is 5% above the country's mean.

Predicting crises. The most important evidence for the scope of this paper is the ability of spreads and credit growth to predict crises. First, Krishnamurthy and Muir (2017) find that conditioning on an episode where credit spreads are below their median value 5 years in a row, the probability of a financial crisis increases by 1.76%. Second, Schularick and Taylor (2012) shows that a one standard deviation increase in credit growth over the preceding 5 years implies an increase in the probability of a crisis of 2.8% over the next year.

Model. I consider an overlapping generation of bank and firm managers living for two periods. In each period a new generation of managers is born and decides information (stage 1) and lending and borrowing (stage 2). In the following period, the shocks are realized, production takes place and firms either repay or default (stage 3). In this period, the old generation of managers receive their payoffs and die, while a new generation is born and the cycle repeats.

I assume that in case of default, firms can not re-enter the economy immediately as it takes one period for the firm to rebuild its productive capacity. This simple constraint can be interpreted as the time needed for new firms to secure funding to cover fixed costs of production or to set up the production process. Define the number of defaulted firms $N_{def,t}$ as the default rate times the number of firms in the economy N_t. Then the number of firms operating in period t is given by $N_t = N_{t-1} - N_{def,t} + N_{def,t-1}$. As illustrated in the previous section, in the presence of limited liability credit booms are followed by a higher default rate, which implies a lower number of productive firms in the economy active in the following period. As a result, booms are followed by a burst, which is consistent with existing evidence.[5]

To relate to the existing evidence on credit cycles, I calibrate one period in the model to represent 5 years in the data. I follow Krishnamurthy and Li (2021) and target an annual unconditional frequency of financial crisis of 4%, which is the average value of the different estimates in the literature. I define a financial crisis as an event in which the output drops below the 20% percentile. I solve for the model equilibrium stage-1 information and stage-2 aggregate quantities and prices for each node in a 15x9 grid of aggregate shock θ_t and number of firms N_t, then I simulate 100,000 periods by drawing from the distribution of θ and interpolating from the grid. I simulate the theoretical moments for both the baseline model without payoff convexity $\psi = 0$ and with payoff convexity $\psi > 0$.

Table G.1 compares the empirical moments to those generated by the model in two different calibrations. First, the baseline model without limited liability is unable to produce systematic movement in spreads or credit before crises or to predict financial crises with movements in spreads or credit. In this model, crises occur only when the economy is hit by negative technological shocks, without any boom-and-bust dynamics. In contrast, the model with limited liability is qualitatively consistent with the evidence. Specifically, crises are systematically preceded by credit booms characterized by an increase in credit and a decline in spreads. Similarly, increases in credit and declines in spreads have predictive power for the probability of future crises. Inattentive managers neglect default risk and over-invest, leading to an overheated economy that will eventually experience a recession in the following period.

[5] While in the framework considered here, booms translate into busts through a credit demand channel, one could think of an alternative setting where the mechanism works instead through a credit supply channel. As shown in the previous section, banks' balance sheets are also impaired after booms as they suffer losses on their loans.

Table G.1: Model and Data Moments

	Data	Model	
		$\psi = 0$	$\psi = .8$
Pre-crisis period (5 years)			
Credit spreads (σ below mean)	0.34	0.00	0.06
Credit/GDP (% above mean)	5	0	7
Predicting crises (5 years)			
Credit spreads (% increase in probability)	1.76	0.00	2.02
Credit/GDP (% increase in probability)	2.8	0.00	4.8

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